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Angular Distribution of Charming $B \to VV$ Decays and Time Evolution Effects

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Abstract

Angular distributions of a B meson decaying into two vector mesons are discussed with emphasis on time evolution effects on the complete set of amplitude bilinears. Time integrated quantities are suggested to observe substantial CP violation in decays with charm quarks in the final state particles. Relations among the nine observables at $t = 0$ are found to be useful for a consistency check of experimentally extracted quantities. Numerical estimates of the nine observables are made using form factor models and the assumption of the factorization hypothesis. Branching ratio asymmetries for $B_u^+ \to D^{*+}\bar{D}^{*0}$ and $B_d \to D^{*+}D^{*-}$ can be as large as -3% and -4%, respectively.

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I. INTRODUCTION

The decays of the B meson into two vector mesons $B \to V_1 + V_2$, either with charm quarks in the final state particles, such as $B \to J/\Psi\,\rho$, or with particles without charm quarks, such as $B \to \rho\, K^*$, have been calculated in many models [1–9]. The time evolution effects in neutral B meson decays are also discussed in [2,7]. In this work, we would like to extend the general discussion on time evolving observables and to emphasize the charming decays in numerical analysis.

One major advantage of analyzing $B \to VV$ decays is that the interference of CP-even and CP-odd final states appear in the angular distributions. These interference terms provide good opportunities to observe CP or T violating effects. Since it is possible to measure all nine observables in certain decays [10], the physically interesting quantities such as β and η can be determined from experiments given sufficient statistics. In addition, relations among the nine observables provide a consistency check for the amplitude bilinears obtained experimentally.

The decay amplitude involves the hadronic matrix element of a B meson decaying to two vector mesons through a weak current, which at present can not be calculated from first principles. Thus, our numerical evaluation of these observables at $t = 0$ is based upon the assumptions of factorization and no final state interactions and form factor models, where the updated Wilson coefficients [11] are used.

This paper is organized as follows: In Section II, we review the observables in the angular distributions of $B \to VV$ decays. In Section III, we derive the time-dependent formulas for the observables and list the complete results in the Appendix. Section IV considers the situation of no time evolution or at $t = 0$. The case with no strong phases is discussed in Section V. Results of single weak amplitude decays are presented in Section VI, wherein CP asymmetries are also extensively discussed. In Section VII, we present the numerical estimation of the nine observables. We summarize this paper in Section VIII.

II. OBSERVABLES AND ANGULAR DISTRIBUTIONS IN $B \to VV$ DECAYS

To extract the CP-odd and CP-even or T-odd and T-even components more easily, the angular distribution is often written in the transversity basis. Let us define the amplitude of $B \to V_1V_2$ in the rest frame of V_1. According to their polarization combinations, the amplitude can be decomposed into [1]

$$A(B \to V_1V_2) = A_0\, \epsilon^{*L}_{V_1}\epsilon^{*L}_{V_2} - \frac{A_\parallel}{\sqrt{2}}\, \vec{\epsilon}^{\,*T}_{V_1} \cdot \vec{\epsilon}^{\,*T}_{V_2} - i\frac{A_\perp}{\sqrt{2}}\, \vec{\epsilon}^{\,*}_{V_1} \times \vec{\epsilon}^{\,*}_{V_2} \cdot \hat{\mathbf{p}}, \tag{1}$$

and similarly for $\bar{B} \to \bar{V}_1\bar{V}_2$. In Eq. (1), $\vec{\epsilon}_{V_1}$ and $\vec{\epsilon}_{V_2}$ are the unit polarization vectors of V_1 and V_2, respectively. $\hat{\mathbf{p}}$ is the unit vector along the direction of motion of V_2 in the rest frame of V_1, $\epsilon^{*L}_{V_i} \equiv \vec{\epsilon}^{\,*}_{V_i} \cdot \hat{\mathbf{p}}$ and $\vec{\epsilon}^{\,*T}_{V_i} = \vec{\epsilon}^{\,*}_{V_i} - \epsilon^{*L}_{V_i}\hat{\mathbf{p}}$. It is easy to see that $A_\perp$ is odd under the parity transformation because of the appearance of $\vec{\epsilon}^{\,*}_{V_1} \times \vec{\epsilon}^{\,*}_{V_2} \cdot \hat{\mathbf{p}}$, whereas A_0 and $A_\parallel$ are even.

The nine observables in the squared amplitude A^*A are [10]

$$\begin{aligned}
K_1(t) &= |A_0(t)|^2, & K_4(t) &= Re\left[A_0^*(t)A_\parallel(t)\right], & L_4(t) &= Im\left[A_0^*(t)A_\parallel(t)\right], \\
K_2(t) &= |A_\parallel(t)|^2, & K_5(t) &= Im\left[A_0^*(t)A_\perp(t)\right], & L_5(t) &= Re\left[A_0^*(t)A_\perp(t)\right], \\
K_3(t) &= |A_\perp(t)|^2, & K_6(t) &= Im\left[A_\parallel^*(t)A_\perp(t)\right], & L_6(t) &= Re\left[A_\parallel^*(t)A_\perp(t)\right].
\end{aligned} \tag{2}$$

So we have

$$\begin{aligned}
A^*(t)A(t) = {} & K_1(t)X_1(\Omega) + K_2(t)X_2(\Omega) + K_3(t)X_3(\Omega) \\
& + K_4(t)X_4(\Omega) + L_5(t)X_5(\Omega) + L_6(t)X_6(\Omega) \\
& + L_4(t)Y_4(\Omega) + K_5(t)Y_5(\Omega) + K_6(t)Y_6(\Omega),
\end{aligned}$$

where the quantities $X_i(\Omega)$ and $Y_i(\Omega)$ represent polarizations or polarization correlations of the final vector mesons and Ω stands for the angles of the outgoing particles.

In general, the angular distribution of the decay in the transversity basis can be written as

$$\frac{d^3\Gamma(t)}{d\cos\theta_1 d\cos\theta_2 d\phi} = \sum_i K_i(t) f_i(\theta_1, \theta_2, \phi), \tag{3}$$

where K_i's are the amplitude bilinears that contain the dynamics and generally evolve with time, and $f_i(\theta_1, \theta_2, \phi)$ are the corresponding angular distribution functions.

One can classify the decays into three types of processes according to the properties of the final product particles as follows:

Type I: For the case in which the decays of V_1 and V_2 are both into two pseudoscalar mesons, one can immediately translate the tensor correlations into angular distributions [10]. The normalized angular distribution of the decays $B \to V_1(\to P_1 P_1')\, V_2(\to P_2 P_2')$, where $P_1^{(\prime)}$ and $P_2^{(\prime)}$ denote pseudoscalar mesons, is:

$$\begin{aligned}\frac{1}{\Gamma_0}\frac{d^3\Gamma(t)}{d\cos\theta_1 d\cos\theta_2 d\phi} = \frac{9}{8\pi}\Big\{ & \frac{K_1(t)}{\Gamma_0}\cos^2\theta_1\cos^2\theta_2 + \frac{K_2(t)}{2\Gamma_0}\sin^2\theta_1\sin^2\theta_2\cos^2\phi \\ & + \frac{K_3(t)}{2\Gamma_0}\sin^2\theta_1\sin^2\theta_2\sin^2\phi + \frac{K_4(t)}{2\sqrt{2}\Gamma_0}\sin 2\theta_1\sin 2\theta_2\cos\phi \\ & - \frac{K_5(t)}{2\sqrt{2}\Gamma_0}\sin 2\theta_1\sin 2\theta_2\sin\phi - \frac{K_6(t)}{2\Gamma_0}\sin^2\theta_1\sin^2\theta_2\sin 2\phi\Big\}.\end{aligned} \tag{4}$$

Here θ_1 (θ_2) is the angle between the P_1 (P_2) three-momentum vector in the $V_1(V_2)$ rest frame and the V_1 (V_2) three-momentum vector defined in the B rest frame, and ϕ is the angle between the normals to the planes defined by P_1P_1' and P_2P_2', in the B rest frame. Examples of such decays are $B^+ \to \bar{D}^*(\to \bar{D}^0\pi^0)\rho^+(\to \pi^+\pi^0)$, $B_d \to D^{*-}(\to \bar{D}^0\pi^-)\rho^+(\to \pi^+\pi^0)$, and $B_d \to D^{*-}(\to \bar{D}^0\pi^-)D^{*+}(\to D^0\pi^+)$.

Type II: For the case of the decay $B \to V_1(\to P_1 P_1')\, V_2(\to l^+ l^-)$, suppose we observe that l^- is a right-handed particle and comes out in the direction $\vec{k_2} = (\sin\theta_2\cos\phi, \sin\theta_2\sin\phi, \cos\theta_2)$ and the momentum of P_1, $\vec{k_1} = (\sin\theta_1, 0, \cos\theta_1)$ with angles defined in the same fashion as in the previous type, we have instead the differential angular distribution [10]:

$$\begin{aligned}\frac{1}{\Gamma_0}\frac{d^3\Gamma}{d\cos\theta_1 d\cos\theta_2 d\phi} = \frac{9}{16\pi\Gamma_0}\Big\{ & K_1\cos^2\theta_1\sin^2\theta_2 + \frac{K_2}{2}\left(\sin^2\theta_1\cos^2\theta_2\cos^2\phi + \sin^2\theta_1\sin^2\phi\right) \\ & + \frac{K_3}{2}\left(\sin^2\theta_1\cos^2\theta_2\sin^2\phi + \sin^2\theta_1\cos^2\phi\right) + \frac{K_4}{2\sqrt{2}}\sin 2\theta_1\sin 2\theta_2\cos\phi \\ & - \frac{K_5}{2\sqrt{2}}\sin 2\theta_1\sin 2\theta_2\sin\phi - \frac{K_6}{2}\sin^2\theta_1\sin^2\theta_2\sin 2\phi \\ & + \frac{L_4}{\sqrt{2}}\sin 2\theta_1\sin\theta_2\sin\phi - \frac{L_5}{\sqrt{2}}\sin 2\theta_1\sin\theta_2\cos\phi + \frac{L_6}{2}\sin^2\theta_1\cos\theta_2\Big\}\end{aligned} \tag{5}$$

To obtain the result for the other possible final state with a left-handed outgoing l^-, one only needs to flip the signs of L_4, L_5, and L_6. The muon polarization is equal to the sum of the terms L_4, L_5, L_6 divided by the sum of the other 6 terms. For the case of L_6 it is seen that the polarization does not vanish after integrating over θ_1 and ϕ and so the observation can be made without observing the V_1 decay. Such decay modes are $B_u^+ \to J/\Psi(\to l^+l^-)K^{*+}(\to \pi^0 K^+)$, $B_u^+ \to J/\Psi(\to l^+l^-)\rho^+(\to \pi^+\pi^0)$, $B_d \to J/\Psi(\to l^+l^-)K^*(\to \pi K)$, $B_d \to J/\Psi(\to l^+l^-)\rho^0(\to \pi\pi)$, $B_d \to J/\Psi(\to l^+l^-)\omega(\to \pi^+\pi^-\pi^0)$, $B_s \to J/\Psi(\to l^+l^-)\bar{K}^*(\to \pi\bar{K})$, and $B_d \to J/\Psi(\to l^+l^-)\phi(\to K^+K^-)$. Although ω decays into three pions, they are still correlated so that one can pick the normal direction to the decay plane formed by the three pions in the ω rest frame to define the direction θ_2 and ϕ.

Although the $B \to V(\to PP)V(\to P\gamma)$ modes have a different decay pattern from that of $B \to V(\to PP)V(\to l^+l^-)$, they share the same differential angular distribution, with the direction of l^- in the latter case replaced by that of γ. For instance, for the decay with a right-handed circularly polarized photon in the final state, the angular distribution is the same as Eq. (5). Such examples are $B_u^+ \to D_s^{*+}(\to D_s^+\gamma)\bar{D}^{*0}(\to \bar{D}^0\pi)$, $B_d \to D_s^{*+}(\to D_s^+\gamma)D^{*-}(\to \bar{D}^0\pi^-)$, $B_s \to D_s^{*-}(\to D_s^-\gamma)D^{*+}(\to D^0\pi^+)$. If one does not measure the polarization of the product particles, the angular distribution would be the one by doubling Eq. (5) and eliminating the $L_{4,5,6}$ terms.

Type III: Next we consider the decay $B \to V(\to P\gamma)V(\to P\gamma)$. Since it is experimentally impractical to measure the polarizations of both photons in the final state, we just give here the differential angular distribution with no polarization measured:

$$
\begin{aligned}
\frac{1}{\Gamma_0}\frac{d^3\Gamma}{d\cos\theta_1 d\cos\theta_2 d\phi} = \frac{9}{8\pi\Gamma_0}\Big\{ & K_1 \sin^2\theta_1 \sin^2\theta_2 \\
& + \frac{K_2}{2}\left(\cos^2\theta_1\cos^2\theta_2\cos^2\phi + \cos^2\theta_1\sin^2\phi + \cos^2\theta_2\sin^2\phi + \cos^2\phi\right) \\
& + \frac{K_3}{2}\left(\cos^2\theta_1\cos^2\theta_2\sin^2\phi + \cos^2\theta_1\cos^2\phi + \cos^2\theta_2\cos^2\phi + \sin^2\phi\right) \\
& - \frac{K_4}{2\sqrt{2}}\sin 2\theta_1 \sin 2\theta_2 \cos\phi + \frac{K_5}{2\sqrt{2}}\sin 2\theta_1 \sin 2\theta_2 \sin\phi \\
& + \frac{K_6}{2}\sin^2\theta_1\sin^2\theta_2\sin 2\phi\Big\}
\end{aligned}
\tag{6}
$$

III. TIME EVOLUTION OF THE AMPLITUDE BILINEARS

The time evolution of an arbitrary neutral B meson state $a|B^0(t)\rangle + b|\bar{B}^0(t)\rangle$ is governed by the Schrödinger equation

$$i\frac{d}{dt}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = \mathcal{H}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix}. \tag{7}$$

If we write the mass eigenstates, $|B_{L,H}\rangle$, with eigenvalues $m_{L,H} - \frac{i}{2}\Gamma_{L,H}$ in terms of $|B^0\rangle$ and $|\bar{B}^0\rangle$ as

$$|B_{L,H}\rangle = p\,|B^0\rangle \pm q\,|\bar{B}^0\rangle, \tag{8}$$

then the time evolutions of B^0 and $\bar{B}^0$ are

$$\begin{aligned} |B^0(t)\rangle &= g_+(t)|B^0(0)\rangle + \frac{q}{p}\,g_-(t)|\bar{B}^0(t)\rangle, \\ |\bar{B}^0(t)\rangle &= \frac{p}{q}\,g_-(t)|B^0(0)\rangle + g_+(t)|\bar{B}^0(t)\rangle, \end{aligned} \tag{9}$$

where

$$g_\pm(t) = \frac{1}{2}\left(e^{-im_Lt}e^{-\frac{1}{2}\Gamma_Lt} \pm e^{-im_Ht}e^{-\frac{1}{2}\Gamma_Ht}\right). \tag{10}$$

Suppose $|f_\eta\rangle$ is a state with definite CP property, namely, $CP|f_\eta\rangle = \eta_i|f_\eta\rangle$ for $i = 1, 2, 3$ and $\eta = 0, \|, \perp$, respectively. The CP eigenvalues $\eta_1 = \eta_2 = +1$ and $\eta_3 = -1$. Suppose we write the decay matrix element of B^0 decaying into the final states f_η at time $t = 0$ as

$$A_\eta(0) \equiv \langle f_\eta|B^0(0)\rangle = Y_{CKM}^T\,e^{i\theta_\eta}(T_\eta + P_\eta\,e^{i\phi_w}e^{i\delta_\eta}). \tag{11}$$

Y_{CKM}^T is the overall CKM factors appearing in the amplitudes. θ_η are the factored strong phases of A_η, but only the relative phases are essential. Conventionally, we take $\theta_\perp = 0$. T_η and P_η are the absolute values of two types of amplitudes that differ by a relative weak phase ϕ_w and a relative strong phase δ_η. We will refer to them by "tree" and "penguin" amplitudes, respectively. Similarly, for the CP conjugate mode we have

$$\bar{A}_\eta(0) \equiv \langle \bar{f}_\eta|\bar{B}^0(0)\rangle = {Y_{CKM}^T}^*\,e^{i\theta_\eta}(T_\eta + P_\eta\,e^{-i\phi_w}e^{i\delta_\eta}). \tag{12}$$

Here we may assume that $|f_\eta\rangle$ and $|\bar{f}_\eta\rangle$ are the same state that both $|B^0\rangle$ and $|\bar{B}^0\rangle$ can decay into (*e.g.*, $B^0, \bar{B}^0 \to J/\Psi\, \phi,\; D^{*+}\, D^{*-}$). They can also be conjugate states so that only $|B^0\rangle$ (or $|B^+\rangle$) can decay into $|f_\eta\rangle$ and only $|\bar{B}^0\rangle$ (or $|B^-\rangle$) to $|\bar{f}_\eta\rangle$. According to the time evolution, the decay amplitude at time t would be

$$A_\eta(t) = \langle f_\eta | B^0(t)\rangle = A_\eta(0)\left[g_+(t) + \eta_i\, \lambda_\eta g_-(t)\right], \tag{13}$$

where

$$\lambda_\eta = \frac{q}{p}\frac{{Y^T_{CKM}}^*}{Y^T_{CKM}}\frac{T_\eta + P_\eta\, e^{-i\phi_w} e^{i\delta_\eta}}{T_\eta + P_\eta\, e^{i\phi_w} e^{i\delta_\eta}}. \tag{14}$$

It is convenient to define a phase ϕ by

$$e^{i\phi} \equiv \frac{q}{p}\frac{{Y^T_{CKM}}^*}{Y^T_{CKM}}, \tag{15}$$

and

$$R_\eta \equiv Re\left[\frac{T_\eta + P_\eta\, e^{-i\phi_w} e^{i\delta_\eta}}{T_\eta + P_\eta\, e^{i\phi_w} e^{i\delta_\eta}}\right],\;\; I_\eta \equiv Im\left[\frac{T_\eta + P_\eta\, e^{-i\phi_w} e^{i\delta_\eta}}{T_\eta + P_\eta\, e^{i\phi_w} e^{i\delta_\eta}}\right]. \tag{16}$$

Note that $R_\eta^2 + I_\eta^2 = 1$ if and only if $\delta_\eta\, , \phi_w = 0 \pmod{\pi}$. If either (i) no nontrivial relative weak phase (0 or π), (ii) negligible tree contributions ($T_\eta \simeq 0$), or (iii) negligible penguin contributions ($P_\eta \simeq 0$) happens, then $R_\eta = 1$ and $I_\eta = 0$, apart from a possible overall phase.

With the above definitions, one can get, for example, the time evolving $|A_\eta(t)|^2$ as follows:

$$\begin{aligned}|A_\eta(t)|^2 = |A_\eta(0)|^2 e^{-\Gamma t}\Bigg\{&\frac{1+R_\eta^2+I_\eta^2}{2}\cosh\left(\frac{\Delta\Gamma\, t}{2}\right) + \frac{1-R_\eta^2-I_\eta^2}{2}\cos\left(\Delta m\, t\right)\\ &+\eta_i\left[(R_\eta\cos\phi - I_\eta\sin\phi)\sinh\left(\frac{\Delta\Gamma\, t}{2}\right) - (R_\eta\sin\phi + I_\eta\cos\phi)\sin\left(\Delta m\, t\right)\right]\Bigg\},\end{aligned} \tag{17}$$

where $\Delta m \equiv m_H - m_L$ and $\Delta\Gamma \equiv \Gamma_H - \Gamma_L$.

Similarly, one uses the time evolution for the conjugate mode to get, along with Eq. (12), for example, the corresponding time evolution formulas for $|\bar{A}_\eta(t)|^2$:

$$\begin{aligned}|\bar{A}_\eta(t)|^2 = |A_\eta(0)|^2 e^{-\Gamma t}\Bigg\{&\frac{1+R_\eta^2+I_\eta^2}{2}\cosh\left(\frac{\Delta\Gamma\, t}{2}\right) - \frac{1-R_\eta^2-I_\eta^2}{2}\cos\left(\Delta m\, t\right)\\ &+\eta_i\left[(R_\eta\cos\phi - I_\eta\sin\phi)\sinh\left(\frac{\Delta\Gamma\, t}{2}\right) + (R_\eta\sin\phi + I_\eta\cos\phi)\sin\left(\Delta m\, t\right)\right]\Bigg\}.\end{aligned} \tag{18}$$

A complete list of all the observable amplitude bilinears and their CP conjugates is given in the Appendix.

Before we proceed the discussion, let's define the CP asymmetry parameters, $\zeta_i(t) \equiv K_i(t) - \bar{K}_i(t)$ for $i = 1, 2, 3..., 6$ and $\xi_i(t) \equiv L_i(t) - \bar{L}_i(t)$ for $i = 4, 5, 6$. These nine parameters measure the changes of the amplitude bilinears under the CP transformation. For instance, from Eqs. (17) and (18), we obtain

$$\zeta_1(t) = K_1(0) e^{-\Gamma t} \left[\left(1 - R_0^2 - I_0^2\right) \cos(\Delta m\, t) - 2\left(R_0 \sin\phi + I_0 \cos\phi\right) \sin(\Delta m\, t) \right]. \qquad (19)$$

This relation along with others for $K_{2,3}(t)$ provide information on ϕ given Δm and Γ extracted from other experiments and theoretical estimates of $K_{1,2,3}(0)$, $R_{0,\|,\perp}$ and $I_{0,\|,\perp}$.

IV. CASE I: NO TIME EVOLUTION

If we take $t = 0$ in Eqs. (A1)-(A5) and (A6)-(A10), we get the bilinear formulas for neutral B meson decays at time $t = 0$, or the charged B meson decays. The relations between the conjugate amplitude bilinears and amplitude bilinears are

$$\begin{aligned}
\bar{K}_i &= \left(R_\eta^2 + I_\eta^2\right) K_i, \text{ for } i = 1, 2, 3, \\
\bar{K}_4 &= \left(R_\| R_0 + I_\| I_0\right) K_4 - \left(I_\| R_0 - R_\| I_0\right) L_4, \\
\bar{K}_{5,6} &= \left(R_\perp R_{0,\|} + I_\perp I_{0,\|}\right) K_{5,6} + \left(I_\perp R_{0,\|} - R_\perp I_{0,\|}\right) L_{5,6}, \\
\bar{L}_4 &= \left(R_\| R_0 + I_\| I_0\right) L_4 + \left(I_\| R_0 - R_\| I_0\right) K_4, \\
\bar{L}_{5,6} &= \left(R_\perp R_{0,\|} + I_\perp I_{0,\|}\right) L_{5,6} - \left(I_\perp R_{0,\|} - R_\perp I_{0,\|}\right) K_{5,6},
\end{aligned} \qquad (20)$$

As discussed in the paragraph after Eq. (16), if none of the relative strong and weak phases are trivial, *i.e.*, 0 or π, CP asymmetry exists in the above bilinears. However, if there are no strong phases (including all the factored strong phases and relative phases) but the relative weak phase is nontrivial, then one can simplify the above equations to get $\bar{K}_{1,2,3,4} = K_{1,2,3,4}$, $\bar{L}_{5,6} = L_{5,6}$, $\bar{K}_{5,6} = -K_{5,6}$, and $\bar{L}_4 = -L_4$. This effect is purely due to

that fact that there is a relative weak phase and $Im[A_0^* A_\parallel]$, $Im[A_0^* A_\perp]$, and $Im[A_\parallel^* A_\perp]$ are CP odd quantities [10].

The observation of CP asymmetries in any of the bilinears indicates that nontrivial strong and weak phases are involved in the decay. Therefore, if the relative weak phase within the Standard Model is trivial, that is, effectively only one weak amplitude dominates, then no CP asymmetry will be observed among all the bilinears.

The formulae presented in this section can be applied to $B_u^+ \to D^{*+}\bar{D}^*$, $B_u^+ \to J/\Psi\rho^+$, $B_u^+ \to D_s^{*+}\bar{D}^*$, and $B_u^+ \to J/\Psi K^{*+}$. One can only measure K_{1-6} in the first decay mode because it is a *Type I* decay. There is no nontrivial weak phases in the latter two decays. Therefore, one should not expect to observe CP asymmetries in the observables; but $K_{5,6}$ and L_4 may be nonzero, and provide evidence for strong phases due to final state interactions. The observation of CP asymmetries in such modes indicates new CP violating source from physics beyond the Standard Model.

V. CASE II: NO STRONG PHASES

If there is no strong phases involved in the decays, then $R_\eta^2 + I_\eta^2 = 1$. One can write

$$R_\eta = \cos 2\alpha_\eta, \qquad I_\eta = -\sin 2\alpha_\eta, \tag{21}$$

where α_η is the phase of $T_\eta + P_\eta e^{i\phi_w}$. With Eq. (21) and the definition of the phase ϕ in Eq. (15), one can get the nine CP asymmetry parameters

$$\zeta_i(t) = 2\eta_i\, K_i(0)e^{-\Gamma t}\sin(2\alpha_\eta - \phi)\sin(\Delta m t),\ \text{for}\ i = 1,2,3, \tag{22}$$

$$\begin{aligned}\zeta_4(t) = {} & K_4(0)e^{-\Gamma t}\left[\sin(2\alpha_\parallel - \phi) + \sin(2\alpha_0 - \phi)\right]\sin(\Delta m t) \\ & - L_4(0)e^{-\Gamma t}\left[\cos(2\alpha_\parallel - \phi) - \cos(2\alpha_0 - \phi)\right]\sin(\Delta m t),\end{aligned} \tag{23}$$

$$\begin{aligned}\zeta_5(t) = K_5(0)e^{-\Gamma t}\Bigg\{ & \cosh\left(\frac{\Delta\Gamma t}{2}\right) + \cos(\Delta m t) \\ & + \left[\cos(2\alpha_0 - \phi) - \cos(2\alpha_\perp - \phi)\right]\sinh\left(\frac{\Delta\Gamma t}{2}\right) \\ & - \cos(2\alpha_0 - 2\alpha_\perp)\left[\cosh\left(\frac{\Delta\Gamma t}{2}\right) - \cos(\Delta m t)\right]\Bigg\}\end{aligned}$$

$$
\begin{aligned}
&+L_5(0)e^{-\Gamma t}\Bigg\{\Big[\sin(2\alpha_0-\phi)+\sin(2\alpha_\perp-\phi)\Big]\sinh\left(\frac{\Delta\Gamma t}{2}\right) \\
&\qquad -\sin(2\alpha_0-2\alpha_\perp)\Big[\cosh\left(\frac{\Delta\Gamma t}{2}\right)-\cos(\Delta m t)\Big]\Bigg\}, \qquad (24)
\end{aligned}
$$

$$
\begin{aligned}
\xi_4(t) = {} & L_4(0)e^{-\Gamma t}\Bigg\{\cosh\left(\frac{\Delta\Gamma t}{2}\right)+\cos(\Delta m t) \\
&\qquad +\Big[\cos(2\alpha_0-\phi)+\cos(2\alpha_\parallel-\phi)\Big]\sinh\left(\frac{\Delta\Gamma t}{2}\right) \\
&\qquad +\cos(2\alpha_0-2\alpha_\parallel)\Big[\cosh\left(\frac{\Delta\Gamma t}{2}\right)-\cos(\Delta m t)\Big]\Bigg\} \\
&-K_4(0)e^{-\Gamma t}\Bigg\{\Big[\sin(2\alpha_\parallel-\phi)-\sin(2\alpha_0-\phi)\Big]\sinh\left(\frac{\Delta\Gamma t}{2}\right) \\
&\qquad -\sin(2\alpha_\parallel-2\alpha_0)\Big[\cosh\left(\frac{\Delta\Gamma t}{2}\right)-\cos(\Delta m t)\Big]\Bigg\}, \qquad (25)
\end{aligned}
$$

$$
\begin{aligned}
\xi_5(t) = {} & L_5(0)e^{-\Gamma t}\left[\sin(2\alpha_\perp-\phi)+\sin(2\alpha_0-\phi)\right]\sin(\Delta m t) \\
&+K_5(0)e^{-\Gamma t}\Big[\cos(2\alpha_\perp-\phi)-\cos(2\alpha_0-\phi)\Big]\sin(\Delta m t). \qquad (26)
\end{aligned}
$$

The formulas for $\zeta_6(t)$ and $\xi_6(6)$ can be obtained by replacing "0" in Eq. (24) and (26) by "$\|$". Thus, in principle, one may extract information about the phase combinations $2\alpha_{0,\|,\perp}-\phi$. One should notice that ξ_4 and $\zeta_{5,6}$ can be nonzero at $t=0$, whereas the others are identically zero. Although the assumption of no strong phases is unlikely to be true in charming decays, it may be applied to charmless decays such as $B \to \rho\rho$.

VI. CASE III: NO RELATIVE WEAK PHASE

If there is no relative weak phase in each transversity amplitude, namely, $\phi_w = 0$, then one gets $R_\eta = 1$ and $I_\eta = 0$. This case is equivalent to the cases where only one type of amplitude dominates the decay process. For completeness, we list time evolutions of the nine observables in Tables I and II [1].

So the nine CP asymmetry parameters are

[1]While we agree with [2] in K_{1-6} and $\bar{K}_{1-4}$, our $\bar{K}_{5,6}$ differ from theirs by an overall minus sign.

Bilinear	Time evolution
$K_i(t)$	$K_i(0)e^{-\Gamma t}\left\{\cosh\left(\frac{\Delta\Gamma t}{2}\right)+\eta_i\left[\cos\phi\sinh\left(\frac{\Delta\Gamma t}{2}\right)-\sin\phi\sin\left(\Delta m t\right)\right]\right\}$ for $i=1,2,3,$
$K_4(t)$	$K_4(0)e^{-\Gamma t}\left[\cosh\left(\frac{\Delta\Gamma t}{2}\right)+\cos\phi\sinh\left(\frac{\Delta\Gamma t}{2}\right)-\sin\phi\sin\left(\Delta m t\right)\right]$
$L_4(t)$	$L_4(0)e^{-\Gamma t}\left[\cosh\left(\frac{\Delta\Gamma t}{2}\right)+\cos\phi\sinh\left(\frac{\Delta\Gamma t}{2}\right)-\sin\phi\sin\left(\Delta m t\right)\right]$
$K_{5,6}(t)$	$K_{5,6}(0)e^{-\Gamma t}\cos\left(\Delta m t\right)-L_{5,6}(0)e^{-\Gamma t}\left[\sin\phi\sinh\left(\frac{\Delta\Gamma t}{2}\right)+\cos\phi\sin\left(\Delta m t\right)\right]$
$L_{5,6}(t)$	$L_{5,6}(0)e^{-\Gamma t}\cos\left(\Delta m t\right)+K_{5,6}(0)e^{-\Gamma t}\left[\sin\phi\sinh\left(\frac{\Delta\Gamma t}{2}\right)+\cos\phi\sin\left(\Delta m t\right)\right]$

TABLE I. Time evolutions of observables in the decay of an initially pure B_q meson into a self-conjugate state of two vector mesons.

Bilinear	Time evolution
$\bar{K}_i(t)$	$K_i(0)e^{-\Gamma t}\left\{\cosh\left(\frac{\Delta\Gamma t}{2}\right)+\eta_i\left[\cos\phi\sinh\left(\frac{\Delta\Gamma t}{2}\right)+\sin\phi\sin\left(\Delta m t\right)\right]\right\}$ for $i=1,2,3$
$\bar{K}_4(t)$	$K_4(0)e^{-\Gamma t}\left[\cosh\left(\frac{\Delta\Gamma t}{2}\right)+\cos\phi\sinh\left(\frac{\Delta\Gamma t}{2}\right)+\sin\phi\sin\left(\Delta m t\right)\right]$
$\bar{L}_4(t)$	$L_4(0)e^{-\Gamma t}\left[\cosh\left(\frac{\Delta\Gamma t}{2}\right)+\cos\phi\sinh\left(\frac{\Delta\Gamma t}{2}\right)+\sin\phi\sin\left(\Delta m t\right)\right]$
$\bar{K}_{5,6}(t)$	$K_{5,6}(0)e^{-\Gamma t}\cos\left(\Delta m t\right)-L_{5,6}(0)e^{-\Gamma t}\left[-\sin\phi\sinh\left(\frac{\Delta\Gamma t}{2}\right)+\cos\phi\sin\left(\Delta m t\right)\right]$
$\bar{L}_{5,6}(t)$	$L_{5,6}(0)e^{-\Gamma t}\cos\left(\Delta m t\right)+K_{5,6}(0)e^{-\Gamma t}\left[-\sin\phi\sinh\left(\frac{\Delta\Gamma t}{2}\right)+\cos\phi\sin\left(\Delta m t\right)\right]$

TABLE II. Time evolutions of observables in the decay of an initially pure $\bar{B}_q$ meson into a self-conjugate state of two vector mesons.

$$
\begin{aligned}
\zeta_i(t) &= -2\eta_i\, K_i(0)e^{-\Gamma t}\sin\phi\sin(\Delta m t), \text{ for } i=1,2,3,\\
\zeta_4(t) &= -2K_4(0)e^{-\Gamma t}\sin\phi\sin(\Delta m t),\\
\zeta_{5,6}(t) &= -2L_{5,6}(0)e^{-\Gamma t}\sin\phi\sinh\left(\frac{\Delta\Gamma t}{2}\right),\\
\xi_4(t) &= -2L_4(0)e^{-\Gamma t}\sin\phi\sin(\Delta m t),\\
\xi_{5,6}(t) &= 2K_{5,6}(0)e^{-\Gamma t}\sin\phi\sinh\left(\frac{\Delta\Gamma t}{2}\right).
\end{aligned}
\tag{27}
$$

These equations hold even if there are nontrivial strong phases.

If we fix the overall strong phases of the transversity amplitudes by the following con-

vention: $A_\perp(0) = |A_\perp(0)|$, $A_0(0) = |A_0(0)|e^{-i\delta_0}$, and $A_\parallel(0) = |A_\parallel(0)|e^{-i\delta_\parallel}$ [2], then $K_{4,5,6}(0)$ and $L_{4,5,6}(0)$ can be rewritten as

$$\begin{aligned} K_4(0) &= \sqrt{K_1(0)K_2(0)}\cos(\delta_0 - \delta_\parallel), & L_4(0) &= \sqrt{K_1(0)K_2(0)}\sin(\delta_0 - \delta_\parallel), \\ K_5(0) &= \sqrt{K_1(0)K_3(0)}\cos(\delta_0), & L_5(0) &= \sqrt{K_1(0)K_3(0)}\sin(\delta_0), \\ K_6(0) &= \sqrt{K_2(0)K_3(0)}\cos(\delta_\parallel), & L_6(0) &= \sqrt{K_2(0)K_3(0)}\sin(\delta_\parallel). \end{aligned} \tag{28}$$

One can readily reach four relations among them:

$$\begin{aligned} K_1(0)K_2(0) &= K_4(0)^2 + L_4(0)^2, & K_2(0)K_3(0) &= K_6(0)^2 + L_6(0)^2, \\ K_3(0)K_1(0) &= K_5(0)^2 + L_5(0)^2, & \frac{L_4(0)}{K_4(0)} &= \frac{L_5(0)K_6(0) - K_5(0)L_6(0)}{K_5(0)K_6(0) + L_5(0)L_6(0)}. \end{aligned} \tag{29}$$

All experimentally measured nine amplitude bilinears should obey the above consistency relations. If the strong phases δ_0 and $\delta_\parallel$ are nontrivial, one could possibly get sizeable $L_{4,5,6}$ that can be observed experimentally. We can then obtain information on the strong phases δ_0, $\delta_\parallel$, the mass difference Δm, the decay width difference $\Delta\Gamma$, and $\sin\phi$ from Eqs. (27). Since some of them share the same time evolution pattern, they also provide a consistency check for the experimental results.

At $t = 0$, there is no CP asymmetry at all. So for charged B decays where one weak amplitude dominates in the Standard Model, one should get the same amplitude bilinears for the particle and its conjugate modes. However, for neutral B decays, the asymmetries develop as time goes on due to the mixing effect. In particular, $\zeta_{1-4}(t)$ and $\xi_4(t)$ have a sinusoidal time dependence, while $\zeta_{5,6}$ and $\xi_{5,6}(t)$ decay exponentially at B_L's decay rate, Γ_L, in the large t limit.

It is, nevertheless, interesting to look at the time integrated quantities for sizeable CP or T violating effects. The particle total decay rate, after time integration, is

$$\int_0^\infty dt\,[K_1(t) + K_2(t) + K_3(t)] = \frac{1}{\Gamma}\Big\{\frac{4}{4-y^2}\,[K_1(0) + K_2(0) + K_3(0)]$$

---

[2]Here we ignore the common weak factor that will be cancelled in all amplitude bilinears.

$$\left(\cos\phi\frac{2y}{4-y^2}-\sin\phi\frac{x}{1+x^2}\right)[K_1(0)+K_2(0)-K_3(0)]\Big\}. \quad (30)$$

Similarly, the time integrated anti-particle total decay rate can be obtained by simply reversing the sign of ϕ in Eq. (30). One can obtain $\sin\phi$ from the asymmetry between the time integrated total rates of conjugate modes and $\cos\phi$ from the untagged analysis. This then eliminates the discrete ambiguity in the angle ϕ.

By integrating Eq. (27) from $t=0$ to $t=\infty$, we find the asymmetries to be

$$\begin{aligned}
\int_0^\infty dt\zeta_i(t) &= -2\eta_i\, K_i(0)\frac{1}{\Gamma}\frac{2x}{1+x^2}\sin\phi, \text{ for } i=1,2,3,\\
\int_0^\infty dt\zeta_4(t) &= -2K_4(0)\frac{1}{\Gamma}\frac{2x}{1+x^2}\sin\phi,\\
\int_0^\infty dt\zeta_{5,6}(t) &= -2L_{5,6}(0)\frac{1}{\Gamma}\frac{2y}{4-y^2}\sin\phi,\\
\int_0^\infty dt\xi_4(t) &= -2L_4(0)\frac{1}{\Gamma}\frac{2x}{1+x^2}\sin\phi,\\
\int_0^\infty dt\xi_{5,6}(t) &= 2K_{5,6}(0)\frac{1}{\Gamma}\frac{2y}{4-y^2}\sin\phi,
\end{aligned} \quad (31)$$

In the above equations, $x\equiv\Delta m/\Gamma$ and $y\equiv\Delta\Gamma/\Gamma$. For B_d, $x=0.73$ and y is negligibly small; for B_s, $x>14.0\,(\mathrm{CL}=95\%)$ and $y<0.67\,(\mathrm{CL}=95\%)$ [12]. From these relations, one can also directly extract $\sin\phi$ given the information about the amplitude bilinears at initial time.

In principle, one can extract information about ϕ and strong phases δ_0, $\delta_\parallel$ either from the time-dependent CP asymmetries ζ's and ξ's or from the integrated asymmetries once the bilinears are determined experimentally or from models. $\sin 2\beta$ has been measured from the mixing-induced CP asymmetry of $B_d\to J/\Psi K_S$ [13]. For $B_d\to J/\Psi K^*(\to\pi K_S)$, the $B_d-\bar{B}_d$ and $K-\bar{K}$ mixings and the CKM factor in the weak decay amplitude also give $\phi=-2\beta$ [14,2]. Therefore, this offers an alternative way of measuring $\sin 2\beta$ through the angular distribution analysis of tagged B_d decays. In addition, the $\cos 2\beta$ dependence in $K_{5,6}$ and $L_{5,6}$ helps resolving the discrete ambiguity of the CKM angle β [2].

For $B_s\to D_s^{*+}D_s^{*-}$ and $B_s\to J/\Psi\phi$, $\phi=2\lambda^2\eta=\mathcal{O}(0.03)$ to an extremely good approximation, where $\lambda=0.22$ is the Cabibbo angle and η is one of the Wolfenstein parameters

[15]. In this case, the CP asymmetries $\zeta_{1,2,3}(t)$ can be used to provide an unambiguous determination of the sign of ϕ, and therefore the sign of η.

The decays that one may apply the results in this section to include: $B_s \to D_s^{*+} D_s^{*-}$, $B_d \to J/\Psi K^*(\to \pi K_S)$, $B_d \to J/\Psi\phi$. The first mode is a *Type III* decay, whereas the latter two are *Type II* decays.

Notice that $\sin\phi$ appears in all CP asymmetries, where ϕ is the phase of mixing and the single CKM factor involved in the decay amplitude. Since the amplitude has only one CKM factor, no CP violation effects would be found in the nine observables at $t = 0$. Yet the mixing will produce differences between the particle and anti-particle decay modes as time goes on. So any observation of the CP asymmetries in such modes indicates CP violation due to mixing and decay.

VII. NUMERICAL CALCULATION

In this section, we apply the factorization hypothesis [16–21] to the calculation of hadronic decay amplitudes. In general, factorization is expected to hold more strongly for color-allowed processes, such as $B_q \to D_s^{*+} \bar{D}_q^*$ with $q \in s, d, u$, though it is doubtful in color-suppressed modes, such as $B_q \to J/\Psi V$ with $(q, V) \in (s, \phi), (d, K^{*0}), (u, K^{*+})$ [22–25,2]. Throughout the calculations, we ignore the strong phases θ_η in Eq. (11) for simplicity but keep the strong phases in the Wilson coefficients [26]. These strong phases may be extracted from experimental data as mentioned in the previous section.

In our calculations, the Wolfenstein parameters are $(\rho, \eta) = (0.18, 0.37)$. The decay constants used are $F_{D^*} = 230MeV$, $F_{D_s^*} = 275MeV$, $F_{J/\Psi} = 394MeV$, $F_{K^*} = 221MeV$, and $F_\phi = 237MeV$. Extracting dominant Wilson coefficients, a_1 and a_2, from experimental decay rates has been performed in [11]. We apply their results to $B^\pm$ and B_d decays with $b \to s$ quark level transitions, i.e., $B_u \to D_s^{*+} \bar{D}^{*0}$, $B_d \to D_s^{*+} D^{*-}$, $B_u \to J/\Psi K^{*+}$, and $B_d \to J/\Psi K^{*0}$. We then extend the results to other decays according to the final state configurations, color-allowed ($B_q \to D_s^{*+} \bar{D}_q^*$ and $B_q \to D^{*+} D^*$) or color-suppressed ($B_q \to$

$J/\Psi V$), charged or neutral. Assuming heavy quark symmetry, we use the $B \to D^*$ decay form factors for the $B \to D_s^*$ transitions.

The bilinears K_i and L_i in the following tables are normalized by dividing with $\Gamma_0 \equiv K_1 + K_2 + K_3$. The branching ratio asymmetry is defined by

$$a_{CP} \equiv \frac{\mathcal{A} - \bar{\mathcal{A}}}{\mathcal{A} + \bar{\mathcal{A}}}, \tag{32}$$

where $\mathcal{A}$ and $\bar{\mathcal{A}}$ are the branching ratios for the particle and antiparticle decays, respectively. In the following tables, we list all nine amplitude bilinears for each mode even if $L_{4,5,6}$ may not be able to be observed from the angular distributions of some of them (Type I decays).

In Tables III and IV, we take the modified BSW (or BSW II) model [17,18] for the form factors in the evaluation of hadronic matrix elements. In Tables V and VI, the Neubert-Stech (NS) model [27] is used. The relativistic light-front (LF) model [28,29] is applied to the calculations in Tables VII and VIII.

We see that in general (1) there are no CP asymmetries for the nine normalized observables at the initial time (yet the CP asymmetries do exist for the unnormalized observables); (2) the branching ratio CP asymmetry is larger in decays involving the $b \to d$ quark level processes because of the relative phase between the CKM factors of two weak amplitudes; and (3) L_4, K_5, and K_6 that involve the imaginary parts of the amplitude bilinears are essentially zero because the tree amplitudes dominates over the penguin contributions in these decays and we ignore possible final state interaction phases.

Although the branching ratios and nine observables may vary as one uses different form factor models, the asymmetries are roughly the same. It is found that $b \to d$ type decays have larger asymmetries than $b \to s$ type ones, as one would expect. From the models we analyze, the asymmetries for $B_u^+ \to D^{*+}\bar{D}^{*0}$ and $B_d \to D^{*+}D^{*-}$ range from -3.29% to -3.53% and from -4.14% to -4.46%, respectively. Unlike the charmless decays where A_0 is the dominant component in the transversity amplitudes, both A_0 and $A_\parallel$ are about the same size. The parity odd component $A_\perp$ is still small in $B \to D^*D^*$ type transitions, but larger in $B \to J/\Psi V$ decays.

VIII. SUMMARY

The decays of a B meson into two vector mesons, which subsequently decay into two lighter particles via CP conserving currents, have a specific pattern in the differential angular distributions. The coefficient of each angular function in the distribution is a bilinear of amplitudes with certain CP properties. The knowledge of these amplitude bilinears can help us observe and understand CP violating effects. In particular, the time evolution of these bilinears in the cases of neutral B meson decays further reveals the information such as the mass and decay width differences and CKM parameters. In situations where we can measure the polarization of the final product particles, all the nine combinations of amplitude bilinears are observable.

Under certain special circumstances, one can find simple relations among the nine observables. Therefore, experimental determination of them is valuable in testing our theoretical assumptions in the calculations, such as the factorization hypothesis and form factor models. The results are particularly simplified when there is only one weak amplitude dominating in the decay process. In such cases, one can test the Standard Model from the CP asymmetries at $t = 0$. The time development of CP asymmetries provides a window for observing CP violations due to mixing effects.

We provide numerical estimates of the observables in 12 sets of charming $B \to VV$ decays using three different form factor models. We find that the results do not depend strongly on the models used. In particular, we find bigger branching ratio asymmetries in $b \to d$ type decays, and those for $B_u^+ \to D^{*+}\bar{D}^{*0}$ and $B_d \to D^{*+}D^{*-}$ are as large as -3% and -4%, respectively.

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APPENDIX: TIME-DEPENDENT AMPLITUDE BILINEARS

The time evolutions of the amplitude bilinears are as follows:

$$
\begin{aligned}
|A_\eta(t)|^2 = |A_\eta(0)|^2 e^{-\Gamma t} \Bigg\{ & \frac{1 + R_\eta^2 + I_\eta^2}{2} \cosh\left(\frac{\Delta\Gamma\, t}{2}\right) + \frac{1 - R_\eta^2 - I_\eta^2}{2} \cos\left(\Delta m\, t\right) \\
& + \eta_i \left[(R_\eta \cos\phi - I_\eta \sin\phi) \sinh\left(\frac{\Delta\Gamma\, t}{2}\right) - (R_\eta \sin\phi + I_\eta \cos\phi) \sin\left(\Delta m\, t\right) \right] \Bigg\},
\end{aligned}
$$

$$
\text{where } i = 1,2,3 \text{ for } \eta = 0, \|, \perp, \text{ respectively, and } \eta_{1,2} = -\eta_3 = 1; \tag{A1}
$$

$$
\begin{aligned}
Re\left[A_0^*(t)A_\|(t)\right] = {} & Re\left[A_0^*(0)A_\|(0)\right] \frac{e^{-\Gamma t}}{2} \times \\
& \Bigg\{ \left[\cosh\left(\frac{\Delta\Gamma\, t}{2}\right) + \cos\left(\Delta m\, t\right)\right] \\
& \quad + (R_\| \cos\phi - I_\| \sin\phi) \sinh\left(\frac{\Delta\Gamma\, t}{2}\right) - (R_\| \sin\phi + I_\| \cos\phi) \sin\left(\Delta m\, t\right) \\
& \quad + (R_0 \cos\phi - I_0 \sin\phi) \sinh\left(\frac{\Delta\Gamma\, t}{2}\right) - (R_0 \sin\phi + I_0 \cos\phi) \sin\left(\Delta m\, t\right) \\
& \quad + (R_\| R_0 + I_\| I_0) \left[\cosh\left(\frac{\Delta\Gamma\, t}{2}\right) - \cos\left(\Delta m\, t\right)\right] \Bigg\} \\
& - Im\left[A_0^*(0)A_\|(0)\right] \frac{e^{-\Gamma t}}{2} \times \\
& \quad \Bigg\{ (R_\| \sin\phi + I_\| \cos\phi) \sinh\left(\frac{\Delta\Gamma\, t}{2}\right) + (R_\| \cos\phi - I_\| \sin\phi) \sin\left(\Delta m\, t\right) \\
& \quad - (R_0 \sin\phi + I_0 \cos\phi) \sinh\left(\frac{\Delta\Gamma\, t}{2}\right) - (R_0 \cos\phi - I_0 \sin\phi) \sin\left(\Delta m\, t\right) \\
& \quad + (I_\| R_0 - R_\| I_0) \left[\cosh\left(\frac{\Delta\Gamma\, t}{2}\right) - \cos\left(\Delta m\, t\right)\right] \Bigg\};
\end{aligned} \tag{A2}
$$

$$
\begin{aligned}
Im\left[A_0^*(t)A_\|(t)\right] = {} & Im\left[A_0^*(0)A_\|(0)\right] \frac{e^{-\Gamma t}}{2} \times \\
& \Bigg\{ \left[\cosh\left(\frac{\Delta\Gamma\, t}{2}\right) + \cos\left(\Delta m\, t\right)\right] \\
& \quad + (R_\| \cos\phi - I_\| \sin\phi) \sinh\left(\frac{\Delta\Gamma\, t}{2}\right) - (R_\| \sin\phi + I_\| \cos\phi) \sin\left(\Delta m\, t\right) \\
& \quad + (R_0 \cos\phi - I_0 \sin\phi) \sinh\left(\frac{\Delta\Gamma\, t}{2}\right) - (R_0 \sin\phi + I_0 \cos\phi) \sin\left(\Delta m\, t\right) \\
& \quad + (R_\| R_0 + I_\| I_0) \left[\cosh\left(\frac{\Delta\Gamma\, t}{2}\right) - \cos\left(\Delta m\, t\right)\right] \Bigg\} \\
& + Re\left[A_0^*(0)A_\|(0)\right] \frac{e^{-\Gamma t}}{2} \times \\
& \quad \Bigg\{ (R_\| \sin\phi + I_\| \cos\phi) \sinh\left(\frac{\Delta\Gamma\, t}{2}\right) + (R_\| \cos\phi - I_\| \sin\phi) \sin\left(\Delta m\, t\right)
\end{aligned}
$$

$$
\begin{aligned}
&-(R_0\sin\phi + I_0\cos\phi)\sinh\left(\frac{\Delta\Gamma\, t}{2}\right) - (R_0\cos\phi - I_0\sin\phi)\sin\left(\Delta m\, t\right)\\
&+(I_\parallel R_0 - R_\parallel I_0)\left[\cosh\left(\frac{\Delta\Gamma\, t}{2}\right) - \cos\left(\Delta m\, t\right)\right]\Bigg\}; \qquad \text{(A3)}
\end{aligned}
$$

$$
\begin{aligned}
Re\left[A_0^*(t)A_\perp(t)\right] = &\, Re\left[A_0^*(0)A_\perp(0)\right]\frac{e^{-\Gamma t}}{2}\times\\
&\Bigg\{\left[\cosh\left(\frac{\Delta\Gamma\, t}{2}\right) + \cos\left(\Delta m\, t\right)\right]\\
&\quad -(R_\perp\cos\phi - I_\perp\sin\phi)\sinh\left(\frac{\Delta\Gamma\, t}{2}\right) + (R_\perp\sin\phi + I_\perp\cos\phi)\sin\left(\Delta m\, t\right)\\
&\quad +(R_0\cos\phi - I_0\sin\phi)\sinh\left(\frac{\Delta\Gamma\, t}{2}\right) - (R_0\sin\phi + I_0\cos\phi)\sin\left(\Delta m\, t\right)\\
&\quad -(R_\perp R_0 + I_\perp I_0)\left[\cosh\left(\frac{\Delta\Gamma\, t}{2}\right) - \cos\left(\Delta m\, t\right)\right]\Bigg\}\\
&+Im\left[A_0^*(0)A_\perp(0)\right]\frac{e^{-\Gamma t}}{2}\times\\
&\quad\Bigg\{(R_\perp\sin\phi + I_\perp\cos\phi)\sinh\left(\frac{\Delta\Gamma\, t}{2}\right) + (R_\perp\cos\phi - I_\perp\sin\phi)\sin\left(\Delta m\, t\right)\\
&\quad +(R_0\sin\phi + I_0\cos\phi)\sinh\left(\frac{\Delta\Gamma\, t}{2}\right) + (R_0\cos\phi - I_0\sin\phi)\sin\left(\Delta m\, t\right)\\
&\quad +(I_\perp R_0 - R_\perp I_0)\left[\cosh\left(\frac{\Delta\Gamma\, t}{2}\right) - \cos\left(\Delta m\, t\right)\right]\Bigg\}; \qquad \text{(A4)}
\end{aligned}
$$

$$
\begin{aligned}
Im\left[A_0^*(t)A_\perp(t)\right] = &\, Im\left[A_0^*(0)A_\perp(0)\right]\frac{e^{-\Gamma t}}{2}\times\\
&\Bigg\{\left[\cosh\left(\frac{\Delta\Gamma\, t}{2}\right) + \cos\left(\Delta m\, t\right)\right]\\
&\quad -(R_\perp\cos\phi - I_\perp\sin\phi)\sinh\left(\frac{\Delta\Gamma\, t}{2}\right) + (R_\perp\sin\phi + I_\perp\cos\phi)\sin\left(\Delta m\, t\right)\\
&\quad +(R_0\cos\phi - I_0\sin\phi)\sinh\left(\frac{\Delta\Gamma\, t}{2}\right) - (R_0\sin\phi + I_0\cos\phi)\sin\left(\Delta m\, t\right)\\
&\quad -(R_\perp R_0 + I_\perp I_0)\left[\cosh\left(\frac{\Delta\Gamma\, t}{2}\right) - \cos\left(\Delta m\, t\right)\right]\Bigg\}\\
&-Re\left[A_0^*(0)A_\perp(0)\right]\frac{e^{-\Gamma t}}{2}\times\\
&\quad\Bigg\{(R_\perp\sin\phi + I_\perp\cos\phi)\sinh\left(\frac{\Delta\Gamma\, t}{2}\right) + (R_\perp\cos\phi - I_\perp\sin\phi)\sin\left(\Delta m\, t\right)\\
&\quad +(R_0\sin\phi + I_0\cos\phi)\sinh\left(\frac{\Delta\Gamma\, t}{2}\right) + (R_0\cos\phi - I_0\sin\phi)\sin\left(\Delta m\, t\right)\\
&\quad +(I_\perp R_0 - R_\perp I_0)\left[\cosh\left(\frac{\Delta\Gamma\, t}{2}\right) - \cos\left(\Delta m\, t\right)\right]\Bigg\}. \qquad \text{(A5)}
\end{aligned}
$$

Similar formulas for $Re\left[A_\parallel^*(0)A_\perp(0)\right]$ and $Im\left[A_\parallel^*(0)A_\perp(0)\right]$ can be obtained from Eq. (A4) and (A5) by replacing "0" with "$\|$", respectively.

The time evolution formulas for the CP conjugate amplitude bilinears are:

$$
\begin{aligned}
|\bar{A}_\eta(t)|^2 = |A_\eta(0)|^2 e^{-\Gamma t}\Bigg\{ & \frac{1+R_\eta^2+I_\eta^2}{2}\cosh\left(\frac{\Delta\Gamma\, t}{2}\right) - \frac{1-R_\eta^2-I_\eta^2}{2}\cos\left(\Delta m\, t\right) \\
& +\eta_i\left[\left(R_\eta\cos\phi - I_\eta\sin\phi\right)\sinh\left(\frac{\Delta\Gamma\, t}{2}\right) + \left(R_\eta\sin\phi + I_\eta\cos\phi\right)\sin\left(\Delta m\, t\right)\right]\Bigg\}; \qquad \text{(A6)}
\end{aligned}
$$

$$
\begin{aligned}
Re\left[\bar{A}_0^*(t)\bar{A}_\parallel(t)\right] = & \left[Re\left[A_0^*(0)A_\parallel(0)\right]\left(R_\parallel R_0 + I_\parallel I_0\right) - Im\left[A_0^*(0)A_\parallel(0)\right]\left(I_\parallel R_0 - R_\parallel I_0\right)\right]\frac{e^{-\Gamma t}}{2}\times \\
& \Bigg\{\left[\cosh\left(\frac{\Delta\Gamma\, t}{2}\right) + \cos\left(\Delta m\, t\right)\right] \\
& \quad +\frac{1}{R_\parallel^2+I_\parallel^2}\left[\left(R_\parallel\cos\phi - I_\parallel\sin\phi\right)\sinh\left(\frac{\Delta\Gamma\, t}{2}\right) + \left(R_\parallel\sin\phi + I_\parallel\cos\phi\right)\sin\left(\Delta m\, t\right)\right] \\
& \quad +\frac{1}{R_0^2+I_0^2}\left[\left(R_0\cos\phi - I_0\sin\phi\right)\sinh\left(\frac{\Delta\Gamma\, t}{2}\right) - \left(R_0\sin\phi + I_0\cos\phi\right)\sin\left(\Delta m\, t\right)\right] \\
& \quad +\frac{1}{\left(R_\parallel^2+I_\parallel^2\right)\left(R_0^2+I_0^2\right)}\left(R_\parallel R_0 + I_\parallel I_0\right)\left[\cosh\left(\frac{\Delta\Gamma\, t}{2}\right) - \cos\left(\Delta m\, t\right)\right]\Bigg\} \\
& -\left[Re\left[A_0^*(0)A_\parallel(0)\right]\left(I_\parallel R_0 - R_\parallel I_0\right) + Im\left[A_0^*(0)A_\parallel(0)\right]\left(R_\parallel R_0 + I_\parallel I_0\right)\right]\frac{e^{-\Gamma t}}{2}\times \\
& \quad \Bigg\{\frac{1}{R_\parallel^2+I_\parallel^2}\left[-\left(R_\parallel\sin\phi + I_\parallel\cos\phi\right)\sinh\left(\frac{\Delta\Gamma\, t}{2}\right) + \left(R_\parallel\cos\phi - I_\parallel\sin\phi\right)\sin\left(\Delta m\, t\right)\right] \\
& \quad -\frac{1}{R_0^2+I_0^2}\left[-\left(R_0\sin\phi + I_0\cos\phi\right)\sinh\left(\frac{\Delta\Gamma\, t}{2}\right) + \left(R_0\cos\phi - I_0\sin\phi\right)\sin\left(\Delta m\, t\right)\right] \\
& \quad -\frac{1}{\left(R_\parallel^2+I_\parallel^2\right)\left(R_0^2+I_0^2\right)}\left(I_\parallel R_0 - R_\parallel I_0\right)\left[\cosh\left(\frac{\Delta\Gamma\, t}{2}\right) - \cos\left(\Delta m\, t\right)\right]\Bigg\}; \qquad \text{(A7)}
\end{aligned}
$$

$$
\begin{aligned}
Im\left[\bar{A}_0^*(t)\bar{A}_\parallel(t)\right] = & \left[Re\left[A_0^*(0)A_\parallel(0)\right]\left(I_\parallel R_0 - R_\parallel I_0\right) + Im\left[A_0^*(0)A_\parallel(0)\right]\left(R_\parallel R_0 + I_\parallel I_0\right)\right]\frac{e^{-\Gamma t}}{2}\times \\
& \Bigg\{\left[\cosh\left(\frac{\Delta\Gamma\, t}{2}\right) + \cos\left(\Delta m\, t\right)\right] \\
& \quad +\frac{1}{R_\parallel^2+I_\parallel^2}\left[\left(R_\parallel\cos\phi - I_\parallel\sin\phi\right)\sinh\left(\frac{\Delta\Gamma\, t}{2}\right) + \left(R_\parallel\sin\phi + I_\parallel\cos\phi\right)\sin\left(\Delta m\, t\right)\right] \\
& \quad +\frac{1}{R_0^2+I_0^2}\left[\left(R_0\cos\phi - I_0\sin\phi\right)\sinh\left(\frac{\Delta\Gamma\, t}{2}\right) + \left(R_0\sin\phi + I_0\cos\phi\right)\sin\left(\Delta m\, t\right)\right] \\
& \quad +\frac{1}{\left(R_\parallel^2+I_\parallel^2\right)\left(R_0^2+I_0^2\right)}\left(R_\parallel R_0 + I_\parallel I_0\right)\left[\cosh\left(\frac{\Delta\Gamma\, t}{2}\right) - \cos\left(\Delta m\, t\right)\right]\Bigg\} \\
& -\left[Re\left[A_0^*(0)A_\parallel(0)\right]\left(R_\parallel R_0 + I_\parallel I_0\right) - Im\left[A_0^*(0)A_\parallel(0)\right]\left(I_\parallel R_0 - R_\parallel I_0\right)\right]\frac{e^{-\Gamma t}}{2}\times
\end{aligned}
$$

$$\left\{\frac{1}{R_\parallel^2+I_\parallel^2}\left[(R_\parallel\sin\phi+I_\parallel\cos\phi)\sinh\left(\frac{\Delta\Gamma\,t}{2}\right)-(R_\parallel\cos\phi-I_\parallel\sin\phi)\sin(\Delta m\,t)\right]\right.$$

$$-\frac{1}{R_0^2+I_0^2}\left[(R_0\sin\phi+I_0\cos\phi)\sinh\left(\frac{\Delta\Gamma\,t}{2}\right)-(R_0\cos\phi-I_0\sin\phi)\sin(\Delta m\,t)\right]$$

$$\left.+\frac{1}{\left(R_\parallel^2+I_\parallel^2\right)(R_0^2+I_0^2)}\left(I_\parallel R_0-R_\parallel I_0\right)\left[\cosh\left(\frac{\Delta\Gamma\,t}{2}\right)-\cos(\Delta m\,t)\right]\right\}; \qquad \text{(A8)}$$

$$Re\left[\bar{A}_0^*(t)\bar{A}_\perp(t)\right]=\left[Re\left[A_0^*(0)A_\perp(0)\right](R_\perp R_0+I_\perp I_0)-Im\left[A_0^*(0)A_\perp(0)\right](I_\perp R_0-R_\perp I_0)\right]\frac{e^{-\Gamma t}}{2}\times$$

$$\left\{\left[\cosh\left(\frac{\Delta\Gamma\,t}{2}\right)+\cos(\Delta m\,t)\right]\right.$$

$$-\frac{1}{R_\perp^2+I_\perp^2}\left[(R_\perp\cos\phi-I_\perp\sin\phi)\sinh\left(\frac{\Delta\Gamma\,t}{2}\right)+(R_\perp\sin\phi+I_\perp\cos\phi)\sin(\Delta m\,t)\right]$$

$$+\frac{1}{R_0^2+I_0^2}\left[(R_0\cos\phi-I_0\sin\phi)\sinh\left(\frac{\Delta\Gamma\,t}{2}\right)+(R_0\sin\phi+I_0\cos\phi)\sin(\Delta m\,t)\right]$$

$$\left.-\frac{1}{(R_\perp^2+I_\perp^2)(R_0^2+I_0^2)}(R_\perp R_0+I_\perp I_0)\left[\cosh\left(\frac{\Delta\Gamma\,t}{2}\right)-\cos(\Delta m\,t)\right]\right\}$$

$$-\left[Re\left[A_0^*(0)A_\perp(0)\right](I_\perp R_0-R_\perp I_0)+Im\left[A_0^*(0)A_\perp(0)\right](R_\perp R_0+I_\perp I_0)\right]\frac{e^{-\Gamma t}}{2}\times$$

$$\left\{\frac{1}{R_\perp^2+I_\perp^2}\left[(R_\perp\sin\phi+I_\perp\cos\phi)\sinh\left(\frac{\Delta\Gamma\,t}{2}\right)-(R_\perp\cos\phi-I_\perp\sin\phi)\sin(\Delta m\,t)\right]\right.$$

$$+\frac{1}{R_0^2+I_0^2}\left[(R_0\sin\phi+I_0\cos\phi)\sinh\left(\frac{\Delta\Gamma\,t}{2}\right)-(R_0\cos\phi-I_0\sin\phi)\sin(\Delta m\,t)\right]$$

$$\left.+\frac{1}{(R_\perp^2+I_\perp^2)(R_0^2+I_0^2)}(I_\perp R_0-R_\perp I_0)\left[\cosh\left(\frac{\Delta\Gamma\,t}{2}\right)-\cos(\Delta m\,t)\right]\right\}; \qquad \text{(A9)}$$

$$Im\left[\bar{A}_0^*(t)\bar{A}_\perp(t)\right]=\left[Re\left[A_0^*(0)A_\perp(0)\right](I_\perp R_0-R_\perp I_0)+Im\left[A_0^*(0)A_\perp(0)\right](R_\perp R_0+I_\perp I_0)\right]\frac{e^{-\Gamma t}}{2}\times$$

$$\left\{\left[\cosh\left(\frac{\Delta\Gamma\,t}{2}\right)+\cos(\Delta m\,t)\right]\right.$$

$$-\frac{1}{R_\perp^2+I_\perp^2}\left[(R_\perp\cos\phi-I_\perp\sin\phi)\sinh\left(\frac{\Delta\Gamma\,t}{2}\right)+(R_\perp\sin\phi+I_\perp\cos\phi)\sin(\Delta m\,t)\right]$$

$$+\frac{1}{R_0^2+I_0^2}\left[(R_0\cos\phi-I_0\sin\phi)\sinh\left(\frac{\Delta\Gamma\,t}{2}\right)+(R_0\sin\phi+I_0\cos\phi)\sin(\Delta m\,t)\right]$$

$$\left.-\frac{1}{(R_\perp^2+I_\perp^2)(R_0^2+I_0^2)}(R_\perp R_0+I_\perp I_0)\left[\cosh\left(\frac{\Delta\Gamma\,t}{2}\right)-\cos(\Delta m\,t)\right]\right\}$$

$$-\left[Re\left[A_0^*(0)A_\perp(0)\right](R_\perp R_0+I_\perp I_0)-Im\left[A_0^*(0)A_\perp(0)\right](I_\perp R_0-R_\perp I_0)\right]\frac{e^{-\Gamma t}}{2}\times$$

$$\left\{\frac{1}{R_\perp^2+I_\perp^2}\left[-(R_\perp\sin\phi+I_\perp\cos\phi)\sinh\left(\frac{\Delta\Gamma\,t}{2}\right)+(R_\perp\cos\phi-I_\perp\sin\phi)\sin(\Delta m\,t)\right]\right.$$

$$+\frac{1}{R_0^2+I_0^2}\left[-(R_0\sin\phi+I_0\cos\phi)\sinh\left(\frac{\Delta\Gamma\,t}{2}\right)+(R_0\cos\phi-I_0\sin\phi)\sin(\Delta m\,t)\right]$$

$$-\frac{1}{\left(R_\perp^2+I_\perp^2\right)\left(R_0^2+I_0^2\right)}\left(I_\perp R_0-R_\perp I_0\right)\left[\cosh\left(\frac{\Delta\Gamma\, t}{2}\right)-\cos\left(\Delta m\, t\right)\right]\Bigg\}. \tag{A10}$$

Similar formulas for $Re\left[\bar{A}_\parallel^*(0)\bar{A}_\perp(0)\right]$ and $Im\left[\bar{A}_\parallel^*(0)\bar{A}_\perp(0)\right]$ can be obtained from Eq. (A9) and (A10) by replacing "0" with "$\|$", respectively.

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Processes	Br($\times 10^{-3}$)	K_1	K_2	K_3	K_4	K_5	K_6	L_4	L_5	L_6
$B_u^+ \to D^{*+}\bar{D}^{*0}$	1.18	0.514	0.415	0.071	-0.462	0	0	0	-0.172	0.191
$B_u^- \to D^{*-}D^{*0}$	1.26	0.514	0.415	0.071	-0.462	0	0	0	-0.172	0.191
$B_u^+ \to J/\Psi\rho^+$	0.0832	0.306	0.413	0.281	-0.356	0	0	0	-0.340	0.293
$B_u^- \to J/\Psi\rho^-$	0.0839	0.306	0.413	0.281	-0.356	0	0	0	-0.340	0.293
$B_d^0 \to D^{*+}D^{*-}$	0.778	0.514	0.415	0.071	-0.462	0	0	0	-0.172	0.191
$\bar{B}_d^0 \to D^{*+}D^{*-}$	0.846	0.514	0.415	0.071	-0.462	0	0	0	-0.172	0.191
$B_d^0 \to J/\Psi\rho^0$	0.0832	0.306	0.413	0.281	-0.356	0	0	0	-0.340	0.293
$\bar{B}_d^0 \to J/\Psi\rho^0$	0.0839	0.306	0.413	0.281	-0.356	0	0	0	-0.340	0.293
$B_s^0 \to D_s^{*-}D^{*+}$	1.05	0.514	0.419	0.067	-0.464	0	0	0	-0.168	0.186
$\bar{B}_s^0 \to D_s^{*+}D^{*-}$	1.07	0.514	0.419	0.067	-0.464	0	0	0	-0.168	0.186
$B_s^0 \to J/\Psi K^{*0}$	0.1076	0.354	0.390	0.256	-0.372	0	0	0	-0.316	0.301
$\bar{B}_s^0 \to J/\Psi\bar{K}^{*0}$	0.1084	0.354	0.390	0.256	-0.372	0	0	0	-0.316	0.301

TABLE III. Charming $B \to VV$ decays involving the $b \to d$ underlying quark processes. BSW II form factors are used in this table. The branching ratio asymmetries of the paired modes are, from top to bottom, -3.29%, -0.38%, -4.14%, -0.38%, -0.97%, and -0.38%, respectively.

Processes	Br($\times 10^{-3}$)	K_1	K_2	K_3	K_4	K_5	K_6	L_4	L_5	L_6
$B_u^+ \to D_s^{*+} D^{*0}$	34.6	0.491	0.438	0.071	-0.464	0	0	0	-0.177	0.187
$B_u^- \to D_s^{*-} \bar{D}^{*0}$	34.5	0.491	0.438	0.071	-0.464	0	0	0	-0.177	0.187
$B_u^+ \to J/\Psi K^{*+}$	1.988	0.358	0.397	0.245	-0.377	0	0	0	-0.312	0.296
$B_u^- \to J/\Psi K^{*-}$	1.987	0.358	0.397	0.245	-0.377	0	0	0	-0.312	0.296
$B_d^0 \to D_s^{*+} D^{*-}$	22.6	0.491	0.438	0.071	-0.464	0	0	0	-0.177	0.187
$\bar{B}_d^0 \to D_s^{*-} D^{*+}$	22.5	0.491	0.438	0.071	-0.464	0	0	0	-0.177	0.187
$B_d^0 \to J/\Psi K^{*0}$	1.988	0.358	0.397	0.245	-0.377	0	0	0	-0.312	0.296
$\bar{B}_d^0 \to J/\Psi \bar{K}^{*0}$	1.987	0.358	0.397	0.245	-0.377	0	0	0	-0.312	0.296
$B_s^0 \to D_s^{*+} D_s^{*-}$	31.28	0.491	0.442	0.068	-0.466	0	0	0	-0.173	0.182
$\bar{B}_s^0 \to D_s^{*+} D_s^{*-}$	31.25	0.491	0.442	0.068	-0.466	0	0	0	-0.173	0.182
$B_s^0 \to J/\Psi \phi$	2.049	0.353	0.413	0.235	-0.382	0	0	0	-0.311	0.288
$\bar{B}_s^0 \to J/\Psi \phi$	2.049	0.353	0.413	0.235	-0.382	0	0	0	-0.311	0.288

TABLE IV. Charming $B \to VV$ decays involving the $b \to s$ underlying quark processes. BSW II form factors are used in this table. The branching ratio asymmetries of the paired modes are, from top to bottom, 0.18%, 0.02%, 0.23%, 0.02%, 0.05%, and 0, respectively.

Processes	Br	K_1	K_2	K_3	K_4	K_5	K_6	L_4	L_5	L_6
$B_u^+ \to D^{*+}\bar{D}^{*0}$	1.24	0.554	0.396	0.050	-0.469	0	0	0	-0.140	0.166
$B_u^- \to D^{*-}D^{*0}$	1.32	0.554	0.396	0.050	-0.469	0	0	0	-0.140	0.166
$B_u^+ \to J/\Psi\rho^+$	0.0957	0.493	0.367	0.141	-0.425	0	0	0	-0.227	0.263
$B_u^- \to J/\Psi\rho^-$	0.0963	0.493	0.367	0.141	-0.425	0	0	0	-0.227	0.263
$B_d^0 \to D^{*+}D^{*-}$	0.81	0.554	0.396	0.050	-0.469	0	0	0	-0.140	0.166
$\bar{B}_d^0 \to D^{*+}D^{*-}$	0.88	0.554	0.396	0.050	-0.469	0	0	0	-0.140	0.166
$B_d^0 \to J/\Psi\rho^0$	0.0957	0.493	0.367	0.141	-0.425	0	0	0	-0.227	0.263
$\bar{B}_d^0 \to J/\Psi\rho^0$	0.0963	0.493	0.367	0.141	-0.425	0	0	0	-0.227	0.263
$B_s^0 \to D_s^{*-}D^{*+}$	1.09	0.552	0.401	0.047	-0.470	0	0	0	-0.137	0.161
$\bar{B}_s^0 \to D_s^{*+}D^{*-}$	1.11	0.552	0.401	0.047	-0.470	0	0	0	-0.137	0.161
$B_s^0 \to J/\Psi K^{*0}$	0.131	0.489	0.383	0.128	-0.433	0	0	0	-0.222	0.250
$\bar{B}_s^0 \to J/\Psi\bar{K}^{*0}$	0.132	0.489	0.383	0.128	-0.433	0	0	0	-0.222	0.250

TABLE V. Charming $B \to VV$ decays involving the $b \to d$ underlying quark processes. The NS model form factors are used in this table. The branching ratio asymmetries of the paired modes are, from top to bottom, -3.29%, -0.30%, -4.14%, -0.30%, -0.97%, and -0.30%, respectively.

Processes	Br	K_1	K_2	K_3	K_4	K_5	K_6	L_4	L_5	L_6
$B_u^+ \to D_s^{*+} D^{*0}$	36.09	0.531	0.420	0.049	-0.472	0	0	0	-0.144	0.162
$B_u^- \to D_s^{*-} \bar{D}^{*0}$	35.96	0.531	0.420	0.049	-0.472	0	0	0	-0.144	0.162
$B_u^+ \to J/\Psi K^{*+}$	2.411	0.484	0.392	0.124	-0.436	0	0	0	-0.220	0.245
$B_u^- \to J/\Psi K^{*-}$	2.410	0.484	0.392	0.124	-0.436	0	0	0	-0.220	0.245
$B_d^0 \to D_s^{*+} D^{*-}$	23.59	0.531	0.420	0.049	-0.472	0	0	0	-0.144	0.162
$\bar{B}_d^0 \to D_s^{*-} D^{*+}$	23.48	0.531	0.420	0.049	-0.472	0	0	0	-0.144	0.162
$B_d^0 \to J/\Psi K^{*0}$	2.4114	0.484	0.392	0.124	-0.436	0	0	0	-0.220	0.245
$\bar{B}_d^0 \to J/\Psi \bar{K}^{*0}$	2.4107	0.484	0.392	0.124	-0.436	0	0	0	-0.220	0.245
$B_s^0 \to D_s^{*+} D_s^{*-}$	32.54	0.529	0.425	0.047	-0.474	0	0	0	-0.141	0.157
$\bar{B}_s^0 \to D_s^{*+} D_s^{*-}$	32.51	0.529	0.425	0.047	-0.474	0	0	0	-0.141	0.157
$B_s^0 \to J/\Psi \phi$	3.166	0.478	0.408	0.114	-0.441	0	0	0	-0.216	0.234
$\bar{B}_s^0 \to J/\Psi \phi$	3.165	0.478	0.408	0.114	-0.441	0	0	0	-0.216	0.234

TABLE VI. Charming $B \to VV$ decays involving the $b \to s$ underlying quark processes. The NS model form factors are used in this table. The branching ratio asymmetries of the paired modes are, from top to bottom, 0.18%, 0.02%, 0.23%, 0.02%, 0.05%, and 0.02%, respectively.

Processes	Br	K_1	K_2	K_3	K_4	K_5	K_6	L_4	L_5	L_6
$B_u^+ \to D^{*+}\bar{D}^{*0}$	1.23	0.557	0.376	0.066	-0.458	0	0	0	-0.158	0.192
$B_u^- \to D^{*-}D^{*0}$	1.32	0.557	0.376	0.066	-0.458	0	0	0	-0.158	0.192
$B_u^+ \to J/\Psi\rho^+$	0.084	0.592	0.334	0.074	-0.445	0	0	0	-0.157	0.209
$B_u^- \to J/\Psi\rho^-$	0.085	0.592	0.334	0.074	-0.445	0	0	0	-0.157	0.209
$B_d^0 \to D^{*+}D^{*-}$	0.81	0.558	0.376	0.066	-0.458	0	0	0	-0.158	0.192
$\bar{B}_d^0 \to D^{*+}D^{*-}$	0.88	0.558	0.376	0.066	-0.458	0	0	0	-0.158	0.192
$B_d^0 \to J/\Psi\rho^0$	0.084	0.592	0.334	0.074	-0.445	0	0	0	-0.157	0.209
$\bar{B}_d^0 \to J/\Psi\rho^0$	0.085	0.592	0.334	0.074	-0.445	0	0	0	-0.157	0.209
$B_s^0 \to D_s^{*-}D^{*+}$	1.10	0.555	0.382	0.063	-0.460	0	0	0	-0.155	0.187
$\bar{B}_s^0 \to D_s^{*+}D^{*-}$	1.19	0.555	0.382	0.063	-0.460	0	0	0	-0.155	0.187
$B_s^0 \to J/\Psi K^{*0}$	0.131	0.536	0.371	0.093	-0.446	0	0	0	-0.186	0.224
$\bar{B}_s^0 \to J/\Psi \bar{K}^{*0}$	0.132	0.536	0.371	0.093	-0.446	0	0	0	-0.186	0.224

TABLE VII. Charming $B \to VV$ decays involving the $b \to d$ underlying quark processes. The LF model form factors are used in this table. The branching ratio asymmetries of the paired modes are, from top to bottom, -3.53%, -0.29%, -4.46%, -0.29%, -1.03%, and -0.29%, respectively.

Processes	Br	K_1	K_2	K_3	K_4	K_5	K_6	L_4	L_5	L_6
$B_u^+ \to D_s^{*+} D^{*0}$	35.42	0.533	0.401	0.067	-0.462	0	0	0	-0.163	0.188
$B_u^- \to D_s^{*-} \bar{D}^{*0}$	35.29	0.533	0.401	0.067	-0.462	0	0	0	-0.163	0.188
$B_u^+ \to J/\Psi K^{*+}$	2.402	0.529	0.381	0.090	-0.449	0	0	0	-0.185	0.218
$B_u^- \to J/\Psi K^{*-}$	2.401	0.529	0.381	0.090	-0.449	0	0	0	-0.185	0.218
$B_d^0 \to D_s^{*+} D^{*-}$	23.06	0.533	0.401	0.067	-0.462	0	0	0	-0.163	0.188
$\bar{B}_d^0 \to D_s^{*-} D^{*+}$	22.94	0.533	0.401	0.067	-0.462	0	0	0	-0.163	0.188
$B_d^0 \to J/\Psi K^{*0}$	2.4025	0.529	0.381	0.090	-0.449	0	0	0	-0.185	0.218
$\bar{B}_d^0 \to J/\Psi \bar{K}^{*0}$	2.4017	0.529	0.381	0.090	-0.449	0	0	0	-0.185	0.218
$B_s^0 \to D_s^{*+} D_s^{*-}$	32.30	0.530	0.407	0.063	-0.464	0	0	0	-0.160	0.183
$\bar{B}_s^0 \to D_s^{*+} D_s^{*-}$	32.27	0.530	0.407	0.063	-0.464	0	0	0	-0.160	0.183
$B_s^0 \to J/\Psi \phi$	2.330	0.496	0.376	0.128	-0.432	0	0	0	-0.220	0.252
$\bar{B}_s^0 \to J/\Psi \phi$	2.329	0.496	0.376	0.128	-0.432	0	0	0	-0.220	0.252

TABLE VIII. Charming $B \to VV$ decays involving the $b \to s$ underlying quark processes. The LF model form factors are used in this table. The branching ratio asymmetries of the paired modes are, from top to bottom, 0.19%, 0.02%, 0.25%, 0.02%, 0.05%, and 0.02%, respectively.